



Data Analysis Round

Good Luck and Clear Skies!

General Instructions

1. The **Data Analysis Round** consists of 1 question on the application of analytical skills on astronomical data.
The allocated time for this round is **90 minutes**.
2. The round will be announced to start by the invigilator (Camp Facilitator).
Please **DO NOT** turn over this page before the start of this round.
3. A countdown timer would be shown on the screen. There are no restrictions to the time budget for each question.
4. There is a total of **80** marks allocated. The marks attributed to each question is marked below the problem statement. The problems are not sorted by difficulty.
5. You will be provided working sheets and **2** graph papers for your solutions. On **EACH** working sheet, answer sheet and graph paper, write down your **Name, IC Number** and **Question Number**. All answers that are to be evaluated must be written on the sheets provided. You are advised to use separated pages for separate questions.
6. **Cross out** sections that you do not want to be evaluated.
7. Use as many mathematical expressions to help the graders better understand your solutions. The graders may not understand your language. If it is necessary to explain something in words, please use short phrases (if possible in English).
8. You are not allowed to leave your working desk without permission. If you need any assistance (malfunctioning calculator, restroom visits, insufficient or missing sheets, etc.), please put up your hand to signal the invigilator.
9. The round would end once the countdown timer rings. At the end of the round, you must stop writing immediately. Sort and put your working sheets and graph paper in one stack. Put papers you do not want to be graded in another stack. You are allowed to keep this question paper.
10. Please remain seated until your papers are collected. You are allowed to leave once all papers are collected.

1. The Waltz of β Pictoris

Astrometric measurements have been crucial in the development of modern astronomy with wide applications ranging from exoplanet hunting to establishing the first rung of the cosmic distance ladder. Astrometry has become extremely precise since measurements were first done in space by the Hipparcos satellite. Now succeeded by Gaia, positions can be pinned down to microarcseconds (μas), allowing precision mapping of more than a billion stars.

Table 1 shows astrometric data of the star β Pictoris ($\alpha = 86.82^\circ$, $\delta = -51.07^\circ$) taken by the Hipparcos mission, where the changes in the coordinates of the star being measured from a reference point.

- (a) You are tasked to determine (i) the distance to β Pic in units of parsecs, and (ii) its proper motion. You are free to analyse the data in whatever method you prefer, but marks will be awarded based on precision. Uncertainty (error) analysis is not necessary.

[65 marks]

In 2009, a hot Jupiter orbiting β Pic was detected through direct imaging. Follow-up monitoring shows that the orbit is edge-on, but the orbital period remains poorly constrained to be between 20 to 26 years due to limited sampling.

Coincidentally, the time interval between the Hipparcos and Gaia missions is 24.25 years, which lie within the uncertainty region of the exoplanet's orbital period. This ensures the orbital phase of the exoplanet during the Gaia observations to be roughly the same as the Hipparcos observations. The combined positional data gives a measurement for the proper motion of β Pic to be

$$\mu_\alpha \cos \delta = 4.94 \pm 0.02 \text{ mas/year} ; \mu_\delta = 83.93 \pm 0.02 \text{ mas/year} . \quad (1)$$

The residual between the proper motion measured by Hipparcos and the 24-year Hipparcos-Gaia baseline is the motion of β Pic in response to the orbital motion of the exoplanet.

- (b) Determine the orientation (position angle) of the exoplanet's orbit.

[15 marks]

Table 1: Astrometric measurements of β Pic measured by Hipparcos

Year	$\Delta\alpha$ (mas)	$\Delta\delta$ (mas)	Year	$\Delta\alpha$ (mas)	$\Delta\delta$ (mas)
1990.01	31.68	-162.95	1991.39	28.72	46.05
1990.08	10.66	-145.84	1991.55	77.83	57.32
1990.16	-3.17	-119.74	1991.65	98.37	44.74
1990.24	-5.17	-86.51	1991.71	104.63	29.11
1990.40	26.05	-35.70	1991.80	98.75	11.49
1990.46	43.39	-27.03	1991.88	81.78	-0.56
1990.56	74.70	-26.04	1991.95	60.98	-3.43
1990.61	88.74	-33.17	1991.95	59.02	-1.98
1990.71	99.65	-52.15	1992.04	32.33	8.71
1990.78	96.21	-68.67	1992.11	13.45	30.39
1990.86	81.79	-81.58	1992.27	7.00	91.81
1990.94	56.93	-85.72	1992.33	17.69	112.82
1991.02	33.26	-77.99	1992.43	45.35	136.09
1991.10	12.33	-56.14	1992.49	62.82	141.38
1991.18	-0.35	-28.78	1992.59	91.14	137.43
1991.25	-0.35	-2.62	1992.89	83.16	81.45
1991.34	13.33	30.38	1993.10	20.61	108.25