# Problem 1 Black Hole in Milky Way

By observational facts, the scientists admit presence of a black-hole at the center of Milky Way.
At the center of Milky Way, a hypothetical black-hole (Sagittarius A*) is located. A star S* is orbiting the black-hole SA*.

In the table 1 the following data is presented: the date and the angular position coordinates $(\alpha;\beta)$ of the star S* at different moments of the observation. The coordinates represent the angular distances of the projection or the star S* in the coordinates system (U, W) , centered on the SA* (see figure 1).

An angular distance of $\varphi = 1\,\text{arcsec}$ corresponds to linear distance in the plane of the sky $d = 41$ light days, therefore to a scale $S_0 = \dfrac{d}{\varphi} = 41\dfrac{\text{light day}}{\text{arcsec}}$.

| | Date (year) | $\alpha(\text{arcsec})$ | $\beta(\text{arcsec})$ |
|---|---|---|---|
| 1 | 1995.222 | 0.117 | - 0.166 |
| 2 | 1997.526 | 0.097 | - 0.189 |
| 3 | 1998.326 | 0.087 | - 0.192 |
| 4 | 1999.041 | 0.077 | - 0.193 |
| 5 | 2000.414 | 0.052 | - 0.183 |
| 6 | 2001.169 | 0.036 | - 0.167 |
| 7 | 2002.831 | - 0.000 | - 0.120 |
| 8 | 2003.584 | - 0.016 | - 0.083 |
| 9 | 2004.165 | - 0.026 | - 0.041 |
| 10 | 2004.585 | - 0.017 | 0.008 |
| 11 | 2004.655 | - 0.004 | 0.014 |
| 12 | 2004.734 | 0.008 | 0.017 |
| 13 | 2004.839 | 0.021 | 0.012 |
| 14 | 2004.936 | 0.037 | 0.009 |
| 15 | 2005.503 | 0.072 | - 0.024 |
| 16 | 2006.041 | 0.088 | - 0.050 |
| 17 | 2007.060 | 0.108 | - 0.091 |

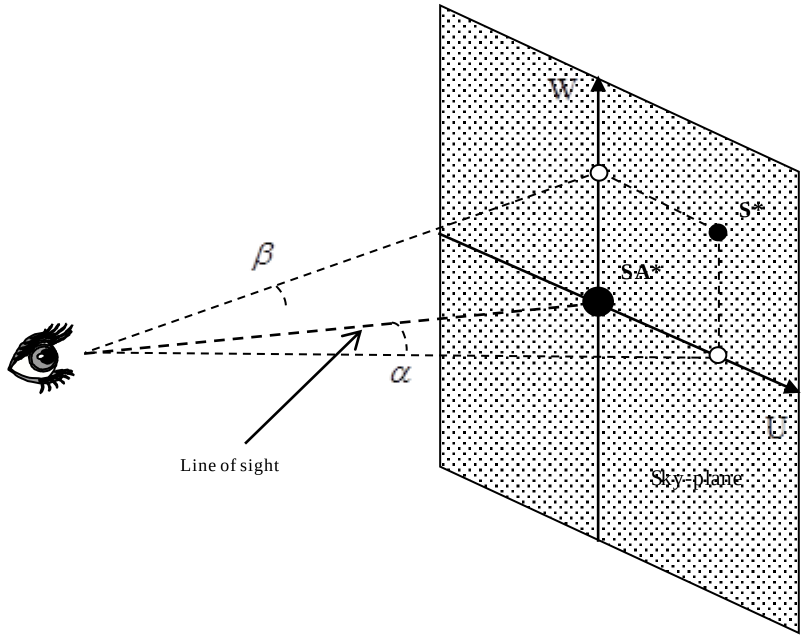



**Fig. 1**

By using the information provided your tasks are:

a) Plot the projection of the trajectory of the star $S^*$ in the plane P (see figure 2). This plane is close to the observer. In this plane, $\varphi = 1\,\text{arcsec}$ corresponds to a linear distance $d_0 = 1200$ mm therefore the scale is $S = \frac{d_0}{\varphi} = 1200\,\frac{mm}{\text{arcsec}}$. You have to use the millimeter graph paper, carbon copy sheet of paper and the transparent sheets for an accurate plot.

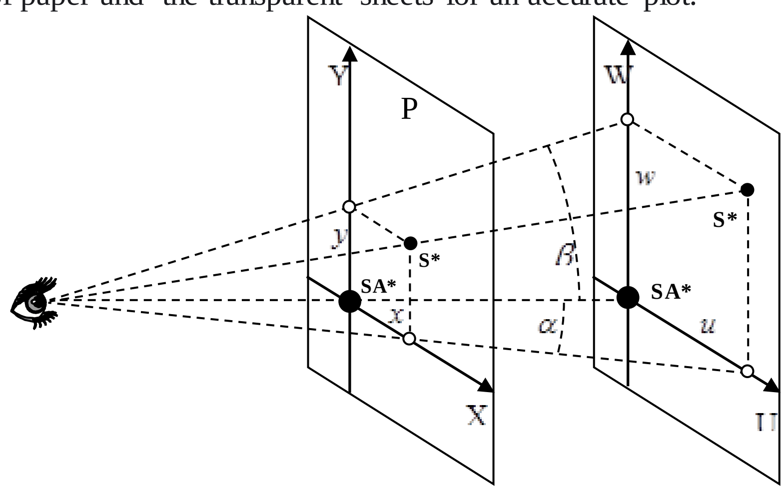



**Fig. 2**

**b)** By using the plot prove that the line of sight is normal to the actual plane of the orbit

**c)** Using your plot find out following elements of the real orbit of star S* around the black hole SA*:

I. $a-$ semi-major axis (in light days units) ; $b-$ small semi-minor axis in (in light days units ); $e-$ eccentricity;

II. $r_{\min}-$ the minimum distance between S* and SA*(in light days units ); $r_{\max}-$ the maximum distance between S* and SA* (in light days units );

III. The distance from the observer to the S*;

IV. the orbiting period of star S* around SA* (obtain the best possible result by taking as many measurements as possible and by taking their arithmetic mean);

V. the total mass of the system "SA* - S*".

Presenting the intermediate and final data in tables is recommended for an accurate evaluation.

$G = 6.67 \cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}}$

## Problem 2. Marking scheme Black Hole in Milky Way

- **a)** **15 points**
- **b)** **5 points**
- **c)** **30 points**
  - I 5 points
  - II 5 points
  - III 5 points
  - IV 10 points
  - V 5 points

| Problem1 | | Punctaj | |
|---|---|---|---|
| a) | Numeric calculations x,y<br>Correct plot on the graph<br>Elyps drawing | 5p<br>5p<br>5p | 15p |
| b) | A≅138 mm<br>B ≅68 mm<br>PC ≅120 mm<br>$c_{calculated} = \sqrt{A^2 - B^2} = 120{,}08\,mm$ | 1p<br>1p<br>1p<br>2p | 5p |
| c I) | $a = \dfrac{41A}{1200} \cong 4{,}715ld$<br>$b = \dfrac{41B}{1200} \cong 2{,}2965ld$ | 2p<br>2p<br>1p | 5p |

| | | | |
|---|---|---|---|
| | $e = \sqrt{1 - \frac{b^2}{a^2}} \cong 0{,}873$ | | |
| c II) | $r_{min} = \frac{d_{\min measured}\, 41}{1200} \cong 0{,}58ld$<br>$r_{max} = \frac{d_{\max measured}\, 41}{1200} \cong 8{,}8ld$ | 2,5p<br>2,5p | 5p |
| c III) | $L = \frac{u}{\alpha} = \frac{w}{\beta}$<br>$L \cong 2318\, ly$ | 3p<br>2p | 5p |
| c IV) | $S_{elipsa} = \pi AB \cong 28560mm^2$<br>Numerical values $\Delta S_{12}, \Delta S_{23}$,..........<br>merical values $\Delta t_{12}, \Delta t_{23}$,...........<br>$T = \frac{S_{elipsa} \Delta t}{\Delta S}$<br>$\overline{T} \cong 15\ ani$ | 2p<br>4p<br>1p<br>2p<br>1p | 10p |
| c V) | $T^2 = \frac{4\pi^2}{k(M_{SA^*} + M_{S^*})} a^3$<br>$M_{SA^*} + M_{S^*} \cong 4{,}6 \cdot 10^{36} kg \cong 2{,}5 \cdot 10^6 M_{\odot}$ | 3p<br>2p | 5p |

# Problem 2 Thermodynamic test

An hypotetical shuttle is launched to investigate the atmosphere (100% $CO_2$) of two extrasolar planets $P_1$ and $P_2$. The atmosphere is in static thermodynamic equilibrium. When the shuttle is near each planet, a radio probe is launched toward respective planet, in vertical direction (in the direction of the planet`s radius). When the radio probe reaches constant velocity, it starts sending values of the pressure of the atmosphere. In Fig. 3.1 is plotted the atmospheric pressure values (in arbitrary units) as function of the time of descent for the planet $P_1$. When the probe touches the surface of planet $P_1$ it sends the value of the temperature $T_0 = 700\,\mathrm{K}$ and the value of the gravitational acceleration $g_0 = 10\,\mathrm{ms}^{-2}$.

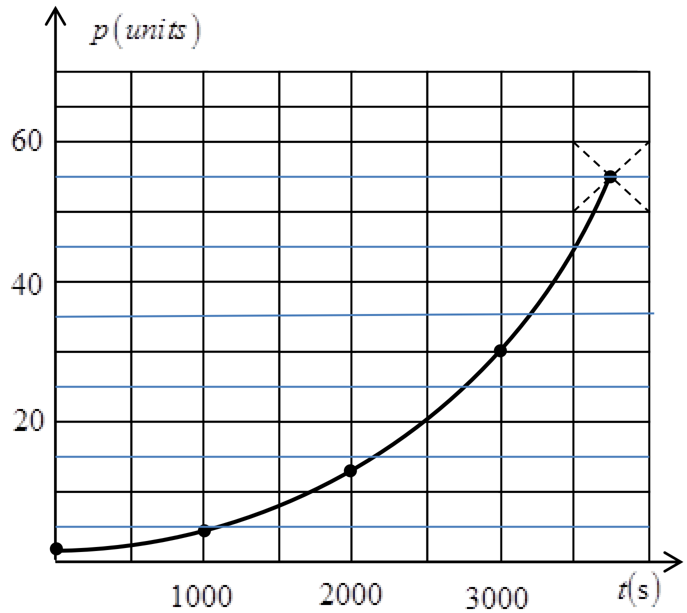



**Fig. 3.1.**

*The gravitational acceleration on each planet is assumed to be constant during uniform descent of the radio probes.*

a) Find the altitude $h_0$ from where the radio probe $R_1$ starts the uniform descent and thus starts the transmitting information.

b) Find the temperature of planet $P_1$ at the altitude $h = 39.6\,\mathrm{km}$. You know: The universal gas constant $R = 8.3\,\mathrm{J/molK}$; the molar mass of $CO_2$, $\mu = 44\,\mathrm{g/mol}$.

c) In Fig. 3.2.was plotted the atmospheric pressure values (in arbitrary units) as a function of time of descent for the planet $P_2$ atmosphere. When the probe touched the surface of the planet $P_2$, it sends the value of the temperature $T_0 = 750\,\mathrm{K}$ and respectively the value of gravitational acceleration $g_0 = 8\,\mathrm{ms}^{-2}$

Draw the following dependency graphs for $p = f(h)$ and $T = f(h)$ in the $CO_2$ atmosphere of the planet $P_2$.

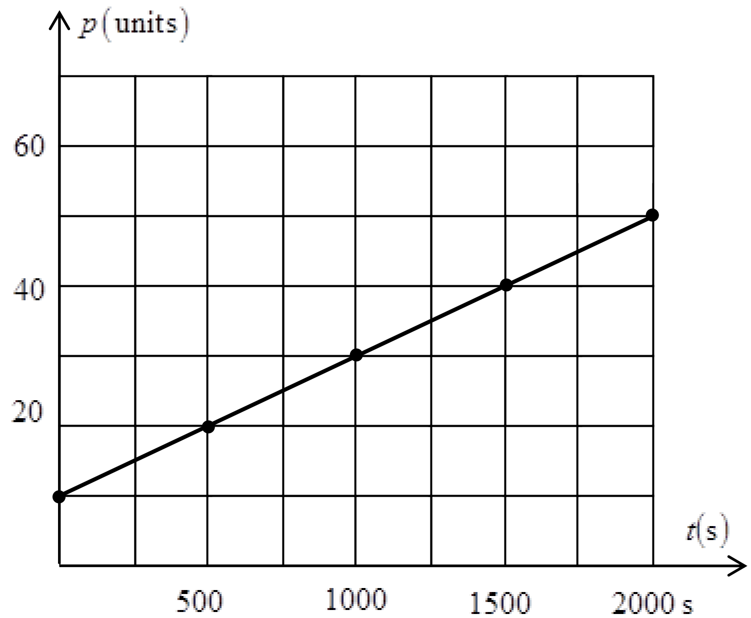



**Fig. 3.2.**

## Problem 2. Marking scheme Thermodynamic test 50 points

| | |
|---|---|
| a. | 10 Points |
| b. | 20 Points |
| c. | 20 points |

**Problem 3 IOAA Observer on an extrasolar planet**

The Sirius star, located in the constellation of Canis Major, is the brightest star in the night sky of the Earth. What the observer's eye sees as a single star is actually a binary star system.

The high brightness of Sirius is a consequence of two facts: its intrinsic luminosity and its proximity to the Earth.

The Mizar multiple star system, in the constellation of Ursa Major, consists of 4 stars seen along the same line of sight from the Earth. Some of these stars form a gravitationally bound system.

Let's assume that an observer (observer A) is located on one of the planets of the Sirius system.

*Determine*:

a) The magnitude of the Sun as seen by observer A ($m_{\text{Sun,Planet}}$).

b) The magnitude of Sirius star system as seen by the observer A. ($m_{SY,\text{Planet}}$)

c) The combined intrinsic luminosity of the Mizar system , $L_{Mizar}$;

d) the average distance between gravitationally bound stars of the Mizar system and Earth, `

e) The geocentric angular distance between Mizar system and Sirius, $\Delta\theta$;

f) The physical distance between the gravitationally bound stars of the Mizar system and the observer A. ($d_{\text{Mizar,Planet}}$)

g) The magnitude of the entire Mizar system as seen by the observer A. ($m_{\text{Mizar,Planet}}$)

Also estimate amount of errors in all your answers.

The following data may be used:

$d_{\text{Sirius},Earth} = 2.6\,\text{pc}$ - the Sirius – Earth distance;

$m_{\text{Sirius},Earth} = -1.46^{\text{m}}$ - the apparent magnitude of Sirius measured from the Earth;

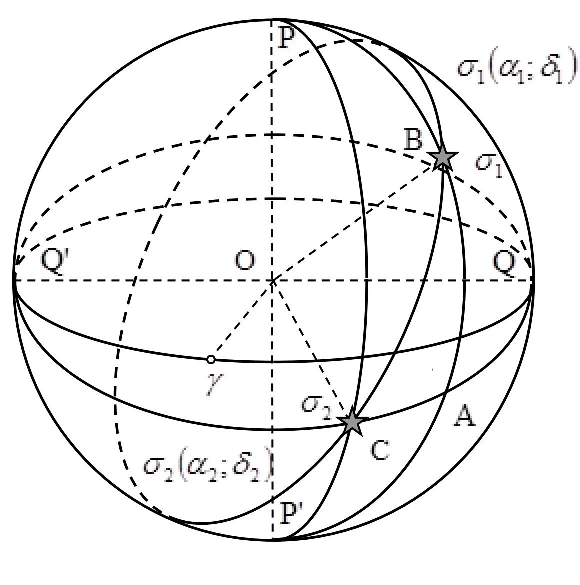
$d_{Sun,Earth} = 1\,\text{AU}$ - the Sun – Earth distance;

$m_{Sun,Earth} = -26{,}78^{\text{m}}$ - the apparent magnitude of the Sun as seen from Earth ;

$d_{\text{Sirius,Planet}} = 10\,\text{AU}$ - distance between Sirius and its planet where the observer A is located;

In the table below information for the stars from the Mizar system as measured from the Earth is given.

| Star number | Name of the star | Apparent magnitude | Parallax (mili arc seconds) |
|---|---|---|---|
| 1 | Alcor | $3.99 \pm 0.01$ | $39.91 \pm 0.13$ |
| 2 | Mizar A | $2.23 \pm 0.01$ | $38.01 \pm 1.71$ |
| 3 | Mizar B | $3.86 \pm 0.01$ | $38.01 \pm 1.71$ |
| 4 | Sidus Ludoviciana | $7.56 \pm 0.01$ | $8 \pm 4$ |



The equatorial coordinates of Mizar system $(\sigma_1)$ and respectively of Sirius $(\sigma_2)$ , located on the heliocentric map are :

$$\alpha_{Mizar} = \alpha_1 = 13^{\mathrm{h}}23^{\mathrm{min}}55.5^{\mathrm{s}};\ \delta_{Mizar} = \delta_1 = 54^{\circ}55'31'';\ \alpha_{\mathrm{Sirius}} = \alpha_2 = 6^{\mathrm{h}}45^{\mathrm{min}};\ \delta_{\mathrm{Sirius}} = \delta_2 = -16^{\circ}43'.$$

Note

$\ln(1-\mathrm{x}) \approx -\mathrm{x}$ for x << 1

$e^x \approx 1+x$ for x << 1

## **Problem 3 .** Marking scheme Observer on an extrasolar star **50 points**

| | |
|---|---|
| a. | 6 Points |
| b. | 6 Points |
| c. | 8 Points |
| d. | 8 Points |
| e. | 8 Points |
| f. | 6 Points |
| g. | 8 Points |

For each part 2 points will be given for calculating the errors

PROBLEMA 3

a) $\lg \dfrac{\dfrac{L_S}{4\pi d_{SE}^2}}{\dfrac{L_S}{4\pi d_{SP}^2}} = \lg\left(\dfrac{d_{SP}}{d_{SE}}\right)^2 = 2\lg\dfrac{d_{SP}}{d_{SE}} = -0{,}4\,(m_{SE} - m_{SP}) \Rightarrow$ 3 p

$d_{SP} \simeq d_{Sy,E} = 2{,}6\ pc = 2{,}6 \cdot 206.265\ UA = 536.289\ UA$

$d_{SE} = 1\ UA$

$\Rightarrow 5\lg\dfrac{d_{SP}}{d_{SE}} = -m_{SE} + m_{SP} \Rightarrow m_{SP} = m_{SE} + 5\lg\dfrac{d_{SP}}{d_{SE}} =$

$= -26{,}78 + 5\lg 536289 = \underline{\underline{1{,}867}}$ 3 p

b) $\lg \dfrac{\dfrac{L_{Sy}}{4\pi d_{Sy,E}^2}}{\dfrac{L_{Sy}}{4\pi d_{Sy,P}^2}} = 2\lg\dfrac{d_{Sy,P}}{d_{Sy,E}} = -0{,}4\,(m_{Sy,E} - m_{Sy,P}) \Rightarrow$ 3 p

$\Rightarrow 5\lg\dfrac{d_{Sy,P}}{d_{Sy,E}} = m_{Sy,P} - m_{SyE} \Rightarrow m_{Sy,P} = m_{SyE} + 5\lg\dfrac{d_{SyP}}{d_{SyE}} =$

$= -1{,}46 + 5\lg\dfrac{10}{2{,}6\cdot 206265} = \underline{\underline{-25{,}14}}$ 3 p

c) $\lg\left(\dfrac{\dfrac{L_K}{4\pi d_{KE}^2}}{\dfrac{L_S}{4\pi d_{SE}^2}}\right) = 2\lg\left(\sqrt{\dfrac{L_K}{L_S}}\cdot\dfrac{d_{SE}}{d_{KE}}\right) = +0{,}4\,(m_{SE} - m_{KE}) \Rightarrow \sqrt{\dfrac{L_K}{L_S}}\cdot\dfrac{d_{SE}}{d_{KE}} =$

$= 10^{0{,}2(m_{SE} - m_{KE})} \Rightarrow \dfrac{L_K}{L_S}\cdot\dfrac{d_{SE}^2}{d_{KE}^2} = 10^{0{,}4(m_{SE} - m_{KE})} \Rightarrow \boxed{L_K = L_S\left(\dfrac{d_{KE}}{d_{SE}}\right)^2 \cdot 10^{0{,}4(m_{SE} - m_{KE})}}$ 1 p

$d_{KE} = \dfrac{d_{SE}}{\operatorname{tg} p''} \simeq \dfrac{d_{SE}}{p''(rad)} = \dfrac{d_{SE}\cdot 18\cdot 36\cdot 10^5}{p\cdot\pi}$

1 p

| | $d_{KE}$ (UA) | $L_K$ ($L_S$) | | $\varepsilon_L$ (2p) |
|---|---|---|---|---|
| Alcor | 25,06<br>5170870 | $13,16 L_S$ | 1p | $0,0867 L_S$ |
| Mizar A | 26,32<br>5.429.346 | $73,36 L_S$ | 1p | $6,62 L_S$ |
| Mizar B | 26,32<br>5.429.346 | $16,35 L_S$ | 1p | $1,471 L_S$ |
| Sidus | 125,06<br>25.796.178 | $12,22 L_S$ | 1p | $4,88 L_S$ |

$$L = L_1 + L_2 + L_3 + L_4 = 115,09\ L_S$$

d) $d_{MASE} = 5.429.346$ UA  8p.  $\varepsilon_d$ 2p. (242114 UA)

$\cos(90^\circ - 54^\circ 55' 31'')\cos(90^\circ + 16^\circ 43') + \sin(90^\circ - 54^\circ 55' 31'')\sin(90^\circ + 16^\circ 43')\cos(\alpha_1 - \alpha_2)$

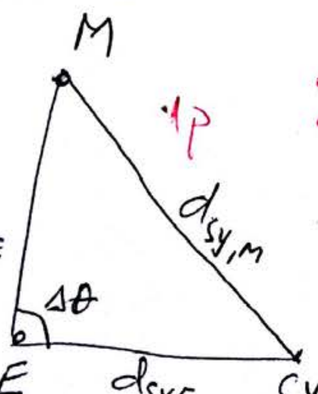
e) $\cos \Delta\theta = \cancel{\cos \Delta\alpha \cdot \cos \Delta\delta = \cos(\alpha_1 - \alpha_2) \cdot \cos(\delta_1 - \delta_2)}$  3p

$\alpha_1 - \alpha_2 = 6^h 38^{min} 55,5^s \simeq 100^\circ \Rightarrow \cos(\alpha_1 - \alpha_2) = -0,1736481777$ ?

$\delta_1 - \delta_2 = 71^\circ 38' 31'' \simeq 71,6^\circ \Rightarrow \cos(\delta_1 - \delta_2) = 0,3156490369$ ?

$\cos \Delta\theta = -0,0548118801 \Rightarrow \cancel{\Delta\theta \simeq 90^\circ}$

$\Delta\theta = 109^\circ$  4p

f) 


1p  2p

$$d_{syM}^2 = d_{MASE}^2 + d_{syE}^2 - 2 d_{MASE} \cdot d_{syE} \cdot \cos\Delta\theta$$

$$= 5429346^2 + (2,6 \cdot 206265)^2 + 2 \cdot 5429346 \cdot 2,6 \cdot 206265 \cdot 0,055 \Rightarrow d_{syM} = 5.485.042 \text{ UA}$$

$= \cancel{26,59}$ 27,26 pc  1p.

$\varepsilon_d$ (242114 UA) 2p.

g) $\underbrace{A, M_{A}, M_B}_{d_1} + \underbrace{SL}_{d_2}$

$$E_1 = \frac{L_1 + L_2 + L_3}{4\pi d_1^2}, \quad E_2 = \frac{L_4}{4\pi d_2^2}$$

$$E_M = \frac{L_1 + L_2 + L_3}{4\pi d_1^2} + \frac{L_4}{4\pi d_2^2}$$

$$\lg \frac{E_{M,A}}{E_{S,A}} = -0{,}4\,(m_{M,A} - m_{S,A})$$

$$\lg \frac{\frac{L_1 + L_2 + L_3}{4\pi d_1^2} + \frac{L_4}{4\pi d_2^2}}{\frac{L_S}{4\pi d_{S,E}^2}} = -0{,}4\,(m_{M,A} - m_{S,A})$$

$$\Rightarrow m_{M,A} \cong 1{,}95^m \pm 0{,}98^m.$$

e) $\cos \Delta\theta = \cos(90° - \delta_1)\cos(90° - \delta_2) + \sin(90° - \delta_1)\sin(90° - \delta_2) \cdot \cos(\alpha_2 - \alpha_1)$

$$\Rightarrow \Delta\theta = 109{,}3°$$